

# Ontology with Likelihood and Typicality of Objects in Concepts

Ching-man Au Yeung and Ho-fung Leung

Department of Computer Science and Engineering  
The Chinese University of Hong Kong  
Shatin, Hong Kong  
{cmaueun, lhf}@cse.cuhk.edu.hk

**Abstract.** Ontologies play an indispensable role in the Semantic Web by specifying the definitions of concepts and individual objects. However, most of the existing methods for constructing ontologies can only specify concepts as crisp sets. However, we cannot avoid encountering concepts that are without clear boundaries, or even vague in meanings. Therefore, existing ontology models are unable to cope with many real cases effectively. With respect to a certain category, certain objects are considered as more representative or typical. Cognitive psychologists explain this by the prototype theory of concepts. This notion should also be taken into account to improve conceptual modeling. While there has been different research attempting to handle vague concepts with fuzzy set theory, formal methods for measuring typicality of objects are still insufficient. We propose a cognitive model of concepts for ontologies, which handles both likelihood (fuzzy membership grade) and typicality of individuals. We also discuss the nature and differences between likelihood and typicality. This model not only enhances the effectiveness of conceptual modeling, but also brings the results of reasoning closer to human thinking. We believe that this research is beneficial to future research on ontological engineering in the Semantic Web.

## 1 Introduction

Ontology [14] is becoming increasingly important and is identified as it plays an important role in enabling information retrieval, information exchange and agent communications [5]. It is also expected to provide semantics to resources on the Web in the emerging Semantic Web [2]. Ontology is usually defined as an explicit specification of conceptualization [13]. Currently, there are several standards for specifying an ontology, such as OWL (Web Ontology Language) [20]. One problem of the existing approaches is that ontologies cannot handle concepts which are vague or without clear boundaries, because concepts in these ontologies are represented as crisp sets of individuals.

However, it is obvious that many concepts we encounter are vague and have no clear boundaries, such as “hot”, “tall” and “far”. In addition, cognitive psychologists also suggest another type of uncertainty in judging membership of objects, which is called “typicality” [24,25]. Typicality reflects how typical or

representative an individual is with respect to a concept [18]. For example, to most English-speaking people, robins are more typical birds than penguins [24]. In this paper, we explain that fuzzy membership grade, which reflects the varying degree of certainty of an individual's membership in concepts (we give this kind of measure the name of *likeliness*), and *typicality*, which reflects the representativeness of an individual with respect to a concept, are two different kinds of measure. When asked whether a penguin is a bird, no one will doubt that the answer is positive, and there is no fuzziness involved. However, many people tend to think that penguin is a less typical bird when compared to other birds, this is the psychological effect that typicality measures.

In this paper, we argue that both likelihood and typicality should be modeled in an ontology to give a clearer picture of an object's membership as well as representativeness with respect to a concept. Modeling typicality in ontology allows reasoning to be more realistic and closer to human thinking. Existing ontology models do not have the mechanisms to determine likelihood and typicality of objects in concepts, and are therefore not able to provide the best and most accurate answers to human users in the reasoning process. With likelihood and typicality, ontologies are able to determine how likely or how typical an object is, and present these results in a way that is more compatible to the expectation of human users. Therefore, we propose a model of concept for constructing ontologies, which is inspired by the Prototype Theory in cognitive psychology, to handle both likelihood and typicality of individual objects.

This paper is structured as follows. Section 2 gives background on ontology and the Prototype Theory in cognitive psychology. Related works are presented in Section 3. Section 4 provides a detail description of the new model of concepts. Section 5 gives an example to illustrate how the model can be used. A discussion of the properties of the model is given in section 6. Finally section 7 mentions future work and concludes the paper.

## 2 Preliminaries

### 2.1 Ontology

Ontology is originally a philosophical discipline which deals with the study of being and existence. The term is borrowed to computer science and defined as an explicit specification of conceptualization [13], which specify the set of concepts that will be used in a particular system as a basis for communication or sharing of information. In particular, ontology is an important component in the Semantic Web [2].

An ontology generally consists of a taxonomy of concepts, a set of relations, a set of individuals (representing real objects), and possibly a set of inference rules for discovering of implicit knowledge [2]. In this paper, we formally define an ontology  $O$  as a four-tuple  $O = (C, P, I, R)$ , where  $C$  is a set of concepts,  $P$  is a set of properties,  $I$  is a set of data instances, representing real objects in the domain of interest, and lastly  $R$  is a set of rules, propositions or axioms that specify the relations between concepts and properties.

Research on ontology is not limited to the specification or construction of ontology, other aspects such as ontology matching [3,33], ontology learning [10,27] and using ontology to assist information retrieval [15,22] are also the foci of ontology researchers. The thorough review paper by Ding [5,6] can be referred to for a more detailed discussion of ontology development.

## 2.2 The Prototype Theory

One of the major areas of research in cognitive psychology is how concepts and categories are represented in the human mind [9]. Until the 1970s, the general view of concept held commonly among psychologists suggested that concepts are defined by singly necessary and jointly sufficient properties. This view is now referred to as the *classical view* [28]. Instances of concepts must meet a set of pre-defined conditions in order to be considered as a member of a concept.

Although the classical view sounds reasonable and intuitive, it has contradicted many empirical findings. Rosch found that people judged different members of a category as varying in representativeness [23,25]. For example, people consider a robin as a better example of bird than others such as ostrich, even though these are all classified as birds. These findings have motivated the development of the *Prototype Theory* of concepts [24]. According to this theory, a concept is represented by a prototype (an abstraction of the concept) in human mind. The prototype of a concept consists of all the salient properties, which are properties that appear frequently in instances classified to this concept. The properties defining the prototype are not limited to necessary properties but also non-necessary ones. This is to model the fact that people use both necessary and non-necessary properties to judge the representativeness of an instance.

The theory explains the existence of varying representativeness of instances by the similarity between the instances and the concept prototype, and use the term *typicality* to refer to the degree of representativeness. It has been found that typicality of an instance can be determined by the number of properties that match between the instance and the prototype. For example, since most birds can fly, the property “*can-fly*” will probably appear in the prototype of the concept “*Bird*”. Hence, birds that can fly will be judged as more *typical* than those that cannot. Moreover, further studies also suggest that properties in the prototype may not be of equal importance [25]. Some of the properties are considered more significant or important to the concept while others are considered less important. Thus, properties are weighted according to their importance in the prototype of a concept.

## 2.3 Likelihood and Typicality of Objects in Concepts

When one learns that concepts have a graded structure (individuals have different membership grades in a concept), one tends to think of fuzzy set theory [34] when they try to model vagueness and uncertainty of concepts, because the theory is a well-known generalization of crisp sets with a characteristic function assigning membership grades to individuals. However, there are in fact differences between likelihood grades and typicality value.

Armstrong, Gleitman and Gleitman [1] point out that typicality effects occur even in some concepts such as *odd number*, which has clear boundaries and definitions. They suggest that one should distinguish membership from prototypicality (typicality). Kamp and Partee [18] also address the distinction between the two, and use  $c^e$  to represent the degree of membership in the extension of a concept ( $e$  stands for goodness of example), and  $c^p$  to represent the degree of typicality ( $p$  stands for prototypicality). While  $c^e$  measures whether or not and to what degree an instance is classified to a concept,  $c^p$  measures how representative or typical is an instance in a concept. It seems that typicality is rather a psychological measure than an objective decision of an individual's membership, because typicality effect is observed even in well-defined concepts.

From a logical perspective, it can also be seen that fuzzy set theory does not capture the essence of the Prototype Theory. As suggested in [24,25], non-necessary properties are involved in determining typicality of instances. Instances that do not possess some of these properties are judged as less typical, but are not judged as non-member. Fuzzy set theory, though a generalization of crisp sets, still requires an element to attain membership greater than zero in each conjuncts in order to attain an overall non-zero membership grade.

In summary, while fuzzy set theory is necessary to model concepts which are vague and has no clear or well-defined boundaries, we need a new mechanism if we want to model typicality of objects in concepts.

### 3 Related Works

Currently, ontologies are constructed by defining concepts and properties using one of the ontology languages. The concepts in these ontologies are interpreted as crisp sets. An individual is either considered as an instance of a concept or it is not. As the theoretical counterpart of ontologies, Description Logics are also restricted to handle crisp concepts [8].

A number of research works apply fuzzy set theory to enhance ontologies. For example, Cross and Cross [4] present fuzzy ontologies to facilitate the retrieval of multilingual documents. Parry [21] introduces fuzzy set theory into ontologies by adding degrees of membership to indicate how likely each term is found in certain locations. Widyantoro and Yen [32] devise a method to construct a fuzzy ontology automatically from a corpus and use the notion of fuzzy narrower term relation to assist querying in a personalized search engine. These works mainly use fuzzy set theory to deal with uncertainty within a taxonomy or a hierarchy of terms, which is less related to definition of concepts and properties.

On the other hand, there are also different works which concern extending Description Logics, the theoretical counterpart of ontologies, to handle fuzziness and uncertainty in concepts. For example, Koller [19] proposes a probabilistic version of Description Logics. On the other hand, Straccia [29,30] combines fuzzy set theory and Description Logics and introduces fuzzy  $\mathcal{ALC}$ , in which concepts are interpreted as fuzzy sets. Straccia also develops a reasoning procedure and an algorithm for deciding satisfiability in fuzzy  $\mathcal{ALC}$ . Others further extend the

expressiveness of fuzzy Description Logics such as by introducing fuzzy hedges, such as “very” and “quite”, as concept modifiers [17].

There are also projects which try to model typicality. For example, [7] describes a frame-based object-centered representation (O.C.R) which incorporates fuzzy set theory to model concepts. The frame slots include information of the typical range of values. With a similar approach, [31] presents an ontology model which represents semantic information about concepts more explicitly. It introduces three characterizations of properties, namely attribute behaviour over time, modality and prototypicality. The model is able to specify whether the value of a property of a concept is typical. However, the framework does not provide mechanisms for calculating an individual object’s typicality in a concept.

Most of the works do not directly address the problem of modeling typicality of instances. The introduction of fuzzy set theory allows handling of fuzzy concepts such as “tall” and “expensive”. These concepts can be represented by a fuzzy set with an appropriate membership function. However, when we consider common concepts such as birds, fishes and furniture, we cannot simply use fuzzy set to model typicality. This is because typicality depends on the properties possessed by the objects, and fuzzy set does not provide the appropriate mechanism to determine typicality.

## 4 Fuzzy Ontology with Likelihood and Typicality

It is clear that the problems that previous works tried to solve by introducing fuzzy set theory into ontologies are quite different from the psychological effect described in the Prototype Theory. Therefore, we are motivated to develop a better model for ontologies which can handle both likelihood grade and typicality of individuals. To handle likelihood, we extend the traditional model of ontologies by using fuzzy set theory. We further extend such model by constructing concept prototypes for the calculation of typicality of individuals.

### 4.1 Concepts and Properties

An ontology is expected to give a formal specification of different concepts in a particular domain. Hence, although we use ideas of the Prototype Theory, we still treat each concept as characterized by a set of necessary properties. This model will be extended to handle both likelihood and typicality. The properties serve as the requirements for being considered as an instance of a concept. A weight is associated with each property in a concept to indicate the importance of that property. For individuals, each of them possesses a set of properties and a value is also associated with each property to indicate the degree to which the individual possesses the property.

**Definition 1.** A *concept*  $x \in C$  is a fuzzy subset of  $I$ , with a membership function  $\mu_x$  assigning each instance  $a \in I$  a membership grade in this concept.

To formally represent concepts and properties, we propose two mathematical notations. Firstly, a *characteristic vector* of a concept is defined as a vector of

real number in the range of 0 to 1, in which each element corresponds to the weight of a different property.

**Definition 2.** A *characteristic vector*  $c_x$  of a concept  $x$  is a vector of real numbers,

$$c_x = (c_{x,1}, c_{x,2}, \dots, c_{x,n}), 0 \leq c_{x,i} \leq 1$$

where  $n$  is the total number of properties.

For an individual, a value of 1 of an element in the characteristic vector means that the property is essential to the concept, while a value of 0 means that the property is not required. Secondly, we define *property vector* of an individual as a vector of real number in the range of 0 to 1, in which each element corresponds to the degree to which the individual possesses a property.

**Definition 3.** The *property vector*  $p_a$  of an individual  $a$  is a vector of real numbers,

$$p_a = (p_{a,1}, p_{a,2}, \dots, p_{a,n}), 0 \leq p_{a,i} \leq 1$$

where  $n$  is the total number of properties.

Concepts in an ontology are generally arranged in a hierarchy such as in OWL [20], and subsumption of concepts are determined by examining whether the set of properties of one concept is a subset of that of another. In this model, we generalize this idea and subsumption of concepts can be determined by comparing the weights in the characteristic vector. For a concept to be considered as subsumed by another concept, it should be characterized at least by all the properties of the latter, and with higher weights for each of these properties.

**Definition 4.** For two concepts  $x$  and  $y$ ,  $x$  is said to be **subsumed by**  $y$ , denoted by  $x \sqsubseteq y$ , if and only if  $c_{x,i} \geq c_{y,i}$  for all  $i = 1, 2, \dots, n$ .

This definition implies two situations that one concept  $x$  can be considered as a sub-concept of another concept  $y$ . In the first case, two concepts are defined by the same set of properties, but  $x$  weights some properties higher than  $y$  does. In the second case,  $x$  has a larger set of defining properties than  $y$ . Both situations are intuitive, because a sub-concept should impose more requirements of properties on an individual than its super-concept.

In addition, we define the notion of sub-concepts, super-concepts, defining properties and possession of properties as follows.

**Definition 5.** If  $x \sqsubseteq y$ , then  $x$  is said to be a **sub-concept** of  $y$ , and  $y$  is said to be a **super-concept** of  $x$ .

**Definition 6.** The set of properties  $P_x$  that includes all properties having a weight greater than zero in the characteristic vector of a concept  $x$  is said to be the set of **defining properties** of  $x$ , or  $x$  is said to be **defined by** the set  $P_x$ .

**Definition 7.** The set of properties  $P_a$  that includes all properties having a degree greater than zero in the property vector of an individual  $a$  is said to be the set of **properties possessed by**  $a$ .

## 4.2 Likeliness of an Individual in a Concept

The first type of uncertainty we want to address is fuzzy membership grade of individuals. We call this degree of membership *likeliness*. The measure of likeliness of an individual determines whether or not and to what degree an individual is classified to a concept according to the defining properties.

**Definition 8.** *In an ontology  $O = (C, P, I, R)$ , **likeliness** of an individual object  $a$  in a concept  $x$  is determined by a function which returns the degree to which  $a$  is considered as an instance of  $x$ :  $\lambda_x : I \rightarrow [0, 1]$ .*

To determine the likeliness of an individual in a concept, a membership function is required. While it is possible to have different functions for likeliness, we argue that likeliness should satisfy the following axioms.

**Axiom 1.** *An individual  $a$  has a degree of likeliness of 1 in a concept  $x$  if and only if  $c_{x,i} > 0 \rightarrow p_{a,i} = 1$  for all  $i = 1, 2, \dots, n$ .*

**Axiom 2.** *An individual  $a$  has a degree of likeliness of 0 in a concept  $x$  if and only if  $c_{x,i} > 0$  and  $p_{a,i} = 0$  for some  $i \in [1, n]$ .*

**Axiom 3.** *For a concept  $x$ , and two individuals  $a$  and  $b$ , if for some  $j$  such that  $c_{x,j} > 0$ ,  $p_{a,j} > p_{b,j}$  and  $p_{a,i} = p_{b,i}$  for all  $i \neq j$ , then  $\lambda_x(a) > \lambda_x(b)$ .*

**Axiom 4.** *For two concepts  $x$  and  $y$ , and an individual  $a$ , if for some  $j$  such that  $c_{x,j} \geq c_{y,j} > 0$ ,  $1 > p_{a,j} > 0$ ,  $c_{x,i} = c_{y,i}$ ,  $p_{a,i} > 0$  for all  $i \neq j$ , then  $\lambda_y(a) \geq \lambda_x(a)$ .*

**Axiom 5.** *For two concepts  $x$  and  $y$ , and an individual  $a$ , if for some  $j$  such that  $c_{x,j} \geq c_{y,j} > 0$ ,  $p_{a,j} = 1$ ,  $c_{x,i} = c_{y,i}$ , and  $p_{a,i} > 0$  for all  $i \neq j$ , then  $\lambda_y(a) = \lambda_x(a)$ .*

Axioms 1 and 2 state the boundary conditions for the degree of likeliness. An individual must possess all the properties with non-zero weight in the characteristic vector in order to be an instance of the concept. To have a likeliness of one, the degree of a property in the property vector should be one whenever that is a defining property of the concept. On the other hand, if the individual does not possess one or more of the defining properties, its likeliness will be zero.

Axioms 3 to 5 state how the degree of likeliness is varied when degrees of possession and property weights change. If one individual possesses a property that the concept assumes non-zero weight to a degree higher than another individual does, then the former will attain a higher degree of likeliness than the latter. This is justified by the fact that the first individual satisfies the requirement to a higher degree. Axiom 4 states that an individual should achieve a higher degree of likeliness in a concept that places lower weights on properties that the individual possesses than another concept that places higher weights on the properties. This axiom is justified because when a property is given higher weight, it is considered as more important and thus there is a more strict requirement on an individual, and therefore the likeliness of an individual is lowered.

Lastly, an exception is described in Axiom 5, which is when the degree of the property in question in the property vector is equal to 1. In this case, since the individual already possesses the property to a full extent, it does not matter how important a property is to the definition of the concept, hence it makes no differences between the degree of likelihood of the individual in the two concepts.

Here, we present a possible function that can be used as the membership function of a concept to determine the degree of likelihood of an individual.

$$\lambda_x(a) = \min_i \{l_i\} \tag{1}$$

where

$$l_i = \begin{cases} p_{a,i} + (1 - c_{x,i}) \times (1 - p_{a,i}) & \text{if } c_{x,i} > 0, p_{a,i} > 0 \\ 0 & \text{if } c_{x,i} > 0, p_{a,i} = 0 \\ 1 & \text{if } c_{x,i} = 0 \end{cases}$$

Since  $p_{a,i}$  is in the range of  $[0,1]$ ,  $\lambda_x(a)$  is also in the range of  $[0,1]$ . The idea of this function is to scale the degrees ( $p_{a,i}$ 's) in the property vector of an individual by using the property weights ( $c_{x,i}$ 's) in the characteristic vector of the concept. The function of likelihood can be used as the membership function of a concept to determine the extent to which an individual object is considered as an instance of a concept:  $\mu_x(a) = \lambda_x(a)$ .

### 4.3 Prototype Vector and Typicality

As suggested by cognitive psychologists [24,18], *typicality* is a measure of how representative or typical is an individual in a particular concept. Typicality is measured based on the number of properties shared by most of the individuals of the concept, which usually include non-necessary properties of a concept [28]. In other words, the characteristic vector alone is not enough to handle typicality because it only contains information of necessary properties of a concept. Therefore, we introduce here a new notation called *prototype vector*.

As typicality of an individual is determined by its similarity to the prototype of a concept [25], we need to first construct a prototype for the concept. According to [28], properties in the prototype “*are salient ones that have a substantial probability of occurring in instances of the concept*”, in other words, weights of the properties in the prototype depend on the saliency of the properties in the instances. In this model, we construct the prototype of a concept based on this idea. However, we rely on weights of properties in the sub-concepts instead of using the saliency of properties. The reason is twofold. Firstly, information is probably be stored in a distributive manner and instances may be scattered in different ontologies. If the weights are dependent on the instances, then the prototypes in different ontologies will tend to be different to a large extent. Moreover, weights of properties in the sub-concepts indicate the importance of the properties, which imply that representative objects will possess properties of higher weights. This also gives us information of the saliency of properties. Therefore, we define the prototype of a concept as follows.

**Definition 9.** The *prototype vector*  $\mathbf{t}_x$  of a concept  $x$  is a vector of real numbers,  $\mathbf{t}_x = (t_{x,1}, t_{x,2}, \dots, t_{x,n}), 0 \leq t_{x,i} \leq 1$ , and is determined by the following equation:

$$\mathbf{t}_x = \frac{1}{|S|} \sum_{s \in S \cup \{x\}} \mathbf{c}_s \quad (2)$$

where  $S$  is the set of sub-concepts of  $x$  as determined by Definition 4.

Typicality is determined by a “weighted feature (property) sum” [28], which means that typicality is reflected by the summation of the weights of the properties that the individual possesses. In our model, this involves first matching the properties in the prototype vector of a concept and the property vector of an individual. We denote the typicality function of a concept by  $\tau_x$ :

**Definition 10.** For an ontology  $O = (C, P, I, R)$ , *typicality* of an individual object  $a$  in a concept  $x$  is determined by a function which returns the degree to  $a$  is considered as a typical instance of  $x$  according to the prototype of  $x$ :  $\tau_x : I \rightarrow [0, 1]$ .

In general, typicality is a function of the prototype vector of the concept and the property vector of the object. We formulate the following axioms which a function for typicality should follow.

**Axiom 6.** An individual  $a$  has a degree of typicality of 1 in a concept  $x$  if and only if  $t_{x,i} > 0 \rightarrow p_{a,i} = 1$  for  $i = 1, 2, \dots, n$ .

**Axiom 7.** An individual  $a$  has a degree of typicality of 0 in a concept  $x$  if and only if  $t_{x,i} > 0 \rightarrow p_{a,i} = 0$  for  $i = 1, 2, \dots, n$ .

**Axiom 8.** For a concept  $x$ , and two individuals  $a$  and  $b$ , if for some  $j$  such that  $t_{x,j} > 0$ ,  $p_{a,j} > p_{b,j} \geq 0$  and  $p_{a,i} = p_{b,i}$  for all  $i \neq j$ , then  $\tau_x(a) > \tau_x(b)$ .

**Axiom 9.** For two concepts  $x$  and  $y$ , and an individual  $a$ , if for some  $j$  such that  $t_{x,j} > t_{y,j} > 0$ ,  $p_{a,j} > 0$  and  $t_{x,i} = t_{y,i}$  for all  $i \neq j$ , then  $\tau_y(a) > \tau_x(a)$ .

Axioms 6 and 7 specify the boundary cases of typicality. According to the Prototype Theory [28], there are two major issues in determining the typicality of an individual in a concept: (1) an individual does not need to possess all the properties in the prototype, and (2) an individual is considered as more typical if it has more properties of the concept prototype. Hence, an individual’s typicality will only be zero when it does not possess any of the properties in the prototype.

Axiom 8 states the influence of degrees in the property vector on typicality. If two individuals possessing the same set of properties, and one possesses the properties which appear in the prototype to a higher degree than the other, then the former will attain a higher typicality than the latter. Moreover, if the first individual possesses more properties in the prototype than the other, the

former individual should attain a higher typicality. This axiom is justified to be in line with the Prototype Theory because in both cases the former individual is considered as more similar to the concept prototype.

The last axiom states that an individual should achieve a higher degree of typicality in a concept that places less weights on properties that the individual possesses than another concept that places more weights on the properties. This is justified because when a property is given more weights, it is more important in the prototype, thus an individual will attain lower typicality in such concept than in another concept which does not consider that property to be that important.

Similar to the discussion on likelihood, we present a possible function for calculating an individual’s typicality in a concept. The typicality of an individual  $a$  of a concept  $x$ , denoted by  $\tau_x(a)$  is given by:

$$\tau_x(a) = \frac{p_a \cdot t_x}{\sum_{i=1}^n t_{x,i}} \tag{3}$$

### 5 Illustrating Example

To illustrate how the proposed model of concepts can measure both likelihood and typicality of objects in concepts, and to provide a more formal and detail demonstration, we present the following example which involves an ontology of birds. <sup>1</sup> Firstly, we assume the following properties in the ontology.

A	Animal	B	Has-Wings	C	Has-Feathers	D	Can-Fly
E	Eat-Seed	F	Has-Curved-Beak	G	Can-Sing	H	Can-Run

We then assume that the following concept are defined using the above properties in the ontology.

- Bird : [A]<sub>1</sub> [B]<sub>1</sub> [C]<sub>1</sub>
- Sparrow : [A]<sub>1</sub> [B]<sub>1</sub> [C]<sub>1</sub> [D]<sub>1</sub> [E]<sub>0.8</sub>
- Parrot : [A]<sub>1</sub> [B]<sub>1</sub> [C]<sub>1</sub> [D]<sub>1</sub> [F]<sub>1</sub>
- Robin : [A]<sub>1</sub> [B]<sub>1</sub> [C]<sub>1</sub> [D]<sub>1</sub> [G]<sub>0.8</sub>
- Ostrich : [A]<sub>1</sub> [B]<sub>1</sub> [C]<sub>1</sub> [H]<sub>0.9</sub>

The above statements define the five concepts (Bird, Sparrow, Parrot, Robin and Ostrich) by using the eight properties listed above. For examples, the concept *Bird* is defined by three properties, namely *is-an-animal*, *has-wings* and *has-feathers*. The numbers written immediately next to each property is the weight of that property. Since there are a total of eight properties, the characteristic vectors of the concepts and the property vectors of the individuals contain eight elements, presumably in the order listed above.

---

<sup>1</sup> This ontology is only for illustration and is not meant to be a precise definition of the birds.

Furthermore, we assume that we have two individuals, a sparrow  $s$  and an ostrich  $o$ , in the ontology representing two real birds. Let the property vectors of the two individuals be

$$\mathbf{p}_s = (1, 1, 1, 0.9, 1, 0, 0, 0), \quad \mathbf{p}_o = (1, 1, 1, 0, 0, 0, 0, 0.8).$$

The property vector of the individual  $s$  indicates that  $s$  possesses properties  $A$ ,  $B$ ,  $C$  and  $E$  to a degree of 1 and property  $D$  to a degree of 0.9, and that of the individual  $o$  indicates that  $o$  possesses properties  $A$ ,  $B$ ,  $C$  to a degree of 1 and property  $H$  to a degree of 0.8. With these information, it is possible to calculate the likeliness and typicality of both individuals. Firstly, we have to obtain the characteristic vectors of the concepts *Sparrow* and *Ostrich*.

$$\mathbf{c}_{\text{Sparrow}} = (1, 1, 1, 1, 0.8, 0, 0, 0), \quad \mathbf{c}_{\text{Ostrich}} = (1, 1, 1, 0, 0, 0, 0, 0.8).$$

Using equation (1), we can then calculate the degree of likeliness of  $s$  in the concept *Sparrow* and that of  $o$  in the concept of *Ostrich*:

$$\begin{aligned} \lambda_{\text{Sparrow}}(s) &= \min(1, 1, 1, 0.9, 1, 1, 1, 1) = 0.9 \\ \lambda_{\text{Ostrich}}(o) &= \min(1, 1, 1, 1, 1, 1, 1, 0.82) = 0.82 \end{aligned}$$

In addition, since both individuals possess all the required properties in the concept *Bird*, it is obvious that their degrees of likeliness in the concept *Bird* are both equal to 1:  $\lambda_{\text{Bird}}(s) = 1$ ,  $\lambda_{\text{Bird}}(o) = 1$ .

The degrees of typicality of the two individuals in the concept *Bird* can be obtained by using the typicality function. Firstly, from the characteristic vectors of the four sub-concepts, the prototype vector of *Bird* can be obtained by using equation (2):

$$\mathbf{t}_{\text{Bird}} = (1, 1, 1, 0.75, 0.25, 0.25, 0.25, 0.225)$$

Using this prototype vector of the concept *Bird*, we can obtain the degrees of typicality of  $s$  and  $o$  by applying the typicality function (4):

$$\tau_{\text{Bird}}(s) = 0.836, \quad \tau_{\text{Bird}}(o) = 0.673.$$

Hence, judging from the typicality of the two individuals, the result suggests that the sparrow  $s$  is a more typical bird than the ostrich  $o$ .

## 6 Discussions

### 6.1 Comparing Likeliness and Typicality

Likeliness is used to model the measure  $c^e$  mentioned by Kamp and Partee [18], which deals with whether or not and to what extent an individual is classified to a particular concept. Typicality, on the other hand, models the measure  $c^p$ , which measures how representative or typical is an individual in a particular concept. As mentioned before, typicality is a less logical and more psychological

measure, because it involves judgement based not only on the necessary and sufficient properties, but involves also non-necessary properties as influenced by its sub-concepts and instances.

Consider the example of birds mentioned in the previous section. Once an individual satisfies the requirements of a concept, it attains a positive value in likelihood. Therefore the two individuals  $s$  and  $o$  both attain a degree of 1 in likelihood in the concept *Bird*. And it is true that they are classified as birds and no one will object to this. However, psychologically, people tend to think of certain birds as more typical. The measure of typicality reflects this phenomenon. As the result of the example suggests, a sparrow is a more typical bird than an ostrich. This is due to the fact that most birds (concepts) in the ontology are defined by the property *can-fly*. As this is a very common property in birds, birds that do not possess this property are likely to be considered as atypical. This result also agrees with findings in cognitive psychology, which suggests that atypical examples are those that are not similar to the prototype of the concept.

From this example, we can see that an individual may attain high degree of likelihood as it fulfils the requirements of the concept, yet still attain a low degree of typicality because it is not similar to the prototype of the concept. The following property summarize this characteristic of the model.

*Property 1.* Assume that two individuals  $a$  and  $b$ , with their property vectors  $\mathbf{p}_a$  and  $\mathbf{p}_b$ , have the same degree of likelihood in a concept  $x$ , i.e.  $\lambda_x(a) = \lambda_x(b)$ . Let  $P_t$  be the set of properties which assume non-zero weights in the prototype vector of  $x$ . If some properties in  $P_t$  is weighted higher in  $\mathbf{p}_a$  than in  $\mathbf{p}_b$  while other properties are weighted the same in both vectors, then we have  $\tau_x(a) > \tau_x(b)$ .

## 6.2 Choosing Between Likelihood and Typicality

Since likelihood and typicality concern different aspects in concepts and categorization, it is also worth to discuss which of the two we should use under different situations. Basically, likelihood is an extension of the traditional way of modeling concepts as crisp sets. As we move on to model vague concepts or concepts without clear boundaries, likelihood provides a measure which more clearly reflects the degree to which the data instances in the ontology are classified to these concepts. For example, we may be interested in “senior employees who have worked in the company for a long period of time”, “flowers with large petals and red in color”, or “restaurants that are close to the railway station and not expensive”. All these concepts – *long period of time*, *large*, *red*, *close*, *expensive* – imply that likelihood is essential in giving us an account of how each data instance in the ontology satisfies these requirements.

On the other hand, typicality provides an alternative mechanism to sort the individuals in a way that is closer to human thinking and psychological belief. Consider again the example of birds, since every individual birds will be classified as birds, it is not possible to sort or rank the individuals by their degrees of likelihood in the concept *Bird*. However, we may sort the individuals based on their typicality ratings, and such order will be similar to what a human user would expect to see. Take searching in the Semantic Web as an example, when

a user searches for birds, it will be a very good idea to present first the data instances that are thought to be more representative or typical. It is also possible that such idea can be further extend to handle more complex concepts.

## 7 Conclusions and Future Work

We presented a novel model of concepts for the construction of ontologies. The model allows both measures of likeliness and typicality of objects in a concept to be represented. We also discuss the nature and differences between likeliness and typicality. A set of axioms which the likeliness function and the typicality function should satisfy is proposed. The model extends traditional ontologies by using fuzzy sets and ideas from cognitive psychology, it provides a mechanism of defining concepts by properties with different weights, and provides a formal method to handle concept prototypes and typicality of objects.

We note that constructing an ontology requires substantial effort, and one challenge of the proposed model is that determining the weights of the properties in the concepts puts extra burden on constructing an ontology. One of our future research directions is to investigate how property weights can be determined more efficiently. For instance, [16] proposes a method for constructing Bayesian networks by combining knowledge from domain expert and information from a small data collection. Similar method may be useful in ontology learning. In addition, the model has much potential in being further developed in different aspects. One of these aspects is context sensitivity. It is mentioned that context is an important issue in knowledge representation [12,11]. Cognitive psychologists also point out that typicality is context-dependent [26]. Hence, we will further investigate how context sensitivity can be incorporated into our ontology model.

## References

1. S. L. Armstrong, L. R. Gleitman, and H. Gleitman. What some concepts might not be. *Cognition*, 13(3):263–308, 1983.
2. T. Berners-Lee, J. Hendler, and O. Lassila. The semantic web. *Sci. Am.*, 284(5):34–43, 2001.
3. V. Cross. Uncertainty in the automation of ontology matching. In *4th International Symposium on Uncertainty Modelling and Analysis*, 2003.
4. V. Cross and C. R. Voss. Fuzzy ontologies for multilingual document exploitation. In *Proceedings of the 1999 Conference of NAFIPS*, pages 392–397, 1999.
5. Ying Ding and Schubert Foo. Ontology research and development part 1 – a review of ontology generation. *Journal of Information Science*, 28(2), 2002.
6. Ying Ding and Schubert Foo. Research and development: Part 2 – a review of ontology mapping and evolving. *Journal of Information Science*, 28(4), 2002.
7. D. Dubois, H. Prade, and J. P. Rossazza. Vagueness, typicality, and uncertainty in class hierarchies. *International Journal of Intelligent Systems*, 6:167–183, 1991.
8. Franz Baader et al., editor. *The Description Logic Handbook: Theory, Implementation, and Applications*. Cambridge University Press, 2003.
9. Kathleen M. Galotti. *Cognitive Psychology In and Out of the Laboratory*. Belmont, CA: Wadsworth, third edition, 2004.

10. Asunción Gómez-Pérez and David Manzano-Macho. An overview of methods and tools for ontology learning from texts. *Knowl. Eng. Rev.*, 19(3):187–212, 2004.
11. D. Grossi, F. Dignum, and J.-J. Ch. Meyer. Contextual taxonomies. In *Proceedings of Fifth International Workshop on Computational Logic in Multi-Agent Systems*, 2004.
12. D. Grossi, F. Dignum, and J.-J. Ch. Meyer. Context in categorization. In *Workshop on Context Representation and Reasoning*, 2005.
13. Thomas R. Gruber. A translation approach to portable ontology specifications. *Knowledge Acquisition*, 5(2):199–220, 1993.
14. Nicola Guarino. Formal ontology and information system. In *Proceedings of the Formal Ontology and Information System*, 1998.
15. R. Guha, R. McCool, and E. Miller. Semantic search. In *WWW '03: Proceedings of the 12th int. conf. on World Wide Web*, pages 700–709, 2003.
16. E. M. Helsen, L. C. van der Gaag, A. J. Feelders, W. L. A. Loeffen, P. L. Geenen, and A. R. W. Elbers. Bringing order into bayesian-network construction. In *Proceedings of Third International Conference on Knowledge Capture*, 2005.
17. Steffen Hölldobler, Tran Dinh Khang, and Hans-Peter Störr. A fuzzy description logic with hedges as concept modifiers. In *IPMU*, 2004.
18. H. Kamp and B. Partee. Prototype theory and compositionality. *Cognition*, 57:129–191, 1995.
19. D. Koller, A. Levy, and A. Pfeffer. P-classic: A tractable probabilistic description logic. In *Proceedings of the 14th National Conference on AI*, pages 390–397, 1997.
20. Deborah L. McGuinness and Frank van Harmelen. OWL web ontology language overview. <http://www.w3.org/TR/owl-features/>, 2004.
21. David Parry. A fuzzy ontology for medical document retrieval. In *CRPIT*, pages 121–126, 2004.
22. C. Rocha, D. Schwabe, and M. de Aragao. A hybrid approach for searching in the semantic web. In *WWW'04*, pages 374–383, 2004.
23. E. H. Rosch. On the internal structure of perceptual and semantic categories. In T. E. More, editor, *Cognitive Development and the Acquisition of Language*. New York: Academic Press, 1973.
24. E. H. Rosch. Cognitive representations of semantic categories. *Journal of Exp. Psy.*, 104:192–233, 1975.
25. Eleanor Rosch and Carolyn B. Mervis. Family resemblances: Studies in the internal structural of categories. *Cognitive Psychology*, 7:573–605, 1975.
26. E. M. Roth and E. J. Shoben. The effect of context on the structure of categories. *Cognitive Psychology*, 15:346–378, 1983.
27. Mehrnosh Shamsfard and Ahmad Abdollahzadeh Barforoush. Learning ontologies from natural language texts. *Int. J. Hum.-Comput. Stud.*, 60(1):17–63, 2004.
28. E. E. Smith and D. L. Medin. *Categories and Concepts*. Harvard University Press, 1981.
29. Umberto Straccia. A fuzzy description logic. In *AAAI*, pages 594–599, 1998.
30. Umberto Straccia. Reasoning within fuzzy description logics. *Journal of Artificial Intelligence Research*, 14:137–166, 2001.
31. V. Tamma and T.J.M. Bench-Capon. An ontology model to facilitate knowledge sharing in multi-agent systems. *Knowledge Engineering Review*, 17(1):41–60, 2002.
32. D. H. Widyantot and J. Yen. Using fuzzy ontology for query refinement in a personalized abstract search engine. In *Proceedings of IFSA and NAFIPS*, 2001.
33. Floris Wiesman and Nico Roos. Domain independent learning of ontology mappings. In *AAMAS*, pages 846–853, 2004.
34. L.A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.